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MICROCRACKING OF CERAMIC MATRIX COMPOSITES  
AT ELEVATED TEMPERATURE

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## 1. Executive Summary

Two research topics have been accomplished through the grant. During the period of March 1995 – March 1998, research was devoted to the microcracking of ceramic matrix composites (CMCs) at elevated temperature. Analysis has been focused on the investigation of crack tip fields of matrix cracks between dissimilar elastic and creeping materials. A brief derivation will be shown in the next section. Depending on the crack tip stress field and strengths of the fiber and the interface, the matrix crack shown in Fig. a may extend into the fiber (fiber failure) or kink along the interface (interface failure).

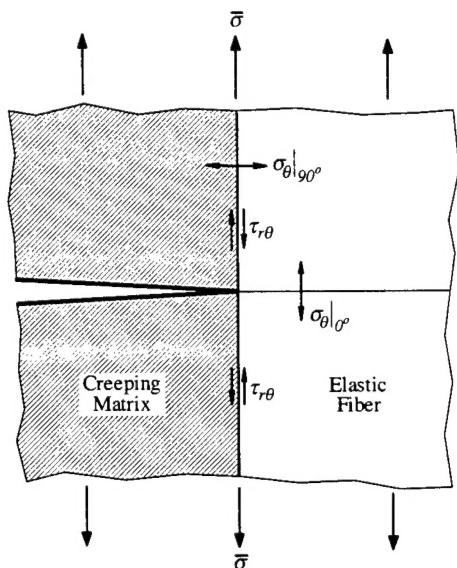


Fig. a Matrix crack in CMCs.

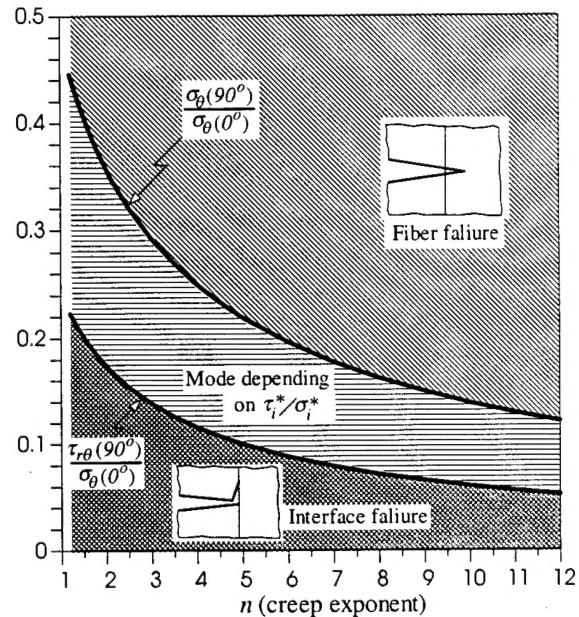


Fig. b Failure mechanism diagram for CMCs at high temperature

Fiber failure will occur if

$$\frac{\sigma_f^*}{\sigma_i^*} < \frac{\sigma_\theta(\theta = 0)}{\sigma_\theta(\theta = 90)} \quad \text{and} \quad \frac{\sigma_f^*}{\tau_i^*} < \frac{\sigma_\theta(\theta = 0)}{\tau_{r\theta}(\theta = 90)}$$

Otherwise, interface failure will prevail. The interface failure can be further divided into debonding and sliding behavior according to

1.  $\frac{\sigma_i^*}{\sigma_f^*} < \frac{\sigma_\theta(\theta = \pi/2)}{\sigma_\theta(\theta = 0)}$  and  $\frac{\sigma_i^*}{\tau_i^*} < \frac{\sigma_\theta(\theta = \pi/2)}{\tau_{r\theta}(\theta = \pi/2)}$  Debonding
2.  $\frac{\tau_i^*}{\sigma_f^*} < \frac{\tau_{r\theta}(\theta = \pi/2)}{\sigma_\theta(\theta = 0)}$  and  $\frac{\tau_i^*}{\sigma_i^*} < \frac{\tau_{r\theta}(\theta = \pi/2)}{\sigma_\theta(\theta = \pi/2)}$  Sliding

where  $\sigma_i^*$  is the interface cohesive strength;

$\sigma_f^*$  is the fiber strength;

$\tau_i^*$  is the interface shear strength.

If the leading-order solution is used to evaluate the crack-tip field, then failure mechanisms and the values of  $\sigma_\theta(\pi/2)/\sigma_\theta(0)$  and  $\tau_{r\theta}(\pi/2)/\sigma_\theta(0)$  for different creep exponent of  $n$  are shown in Fig. b. The impact resulting from the research may be summarized as below:

(1) All of the composite's mechanical characteristics that govern structural performance and life depend upon the constituent properties (fibers, matrix, interfaces). Because the constituents are variables, optimization of the property profiles needed for design and lifing become prohibitively expensive if traditional empirical procedures are used, especially for CMCs under high temperature where creep deformation is prevalent.

(2) The toughness of CMCs at high temperature can be enhanced by achieving crack deflection at the interfaces between fibers and matrix, rather than propagation into the fiber. In this research, among the constituent properties, the critical parameters, creep exponent of the matrix ( $n$ ) and strength ratios, are identified based on mechanics-based models. These parameters provide the designer with the ability to vary interface properties as desired to attain such a goal. A failure mechanism diagram is established to demonstrate the propensity of these failure mechanisms.

During the period March – September, 1998, the work started with research in the area of microfailure of composite laminates with the aim of establishing failure criteria in composites. The study has been focused on the investigation of microstress in a model composite laminate. The model composite laminate consists of well-controlled microgeometry of boron/epoxy or SiC/epoxy layers bonded on an epoxy layer. The composite is reinforced by hundreds of fibers per square inch of cross-sectional area. Since the steep stress gradients occur in the region of few fibers, only eight fibers have been analyzed. The test of the composite laminate under increasing  $\varepsilon_y$  is being performed at Materials Lab. at WPAFB. By varying the bond strength between the fibers and matrix, different initial failure modes, such as interfacial debonding, matrix cracking, and fiber breakage can be initiated under the macroscopic loading. By correlating the analyses with the experimental observations under critical loads, the failure initiation criteria of each failure mode by incorporating the proposed failure hypotheses by are currently evaluated and will be quantitatively established based on stress-based criteria.

## 2. Accomplishments/New Findings

### (A) Crack-Tip Fields for Matrix Cracks between Elatic fiber and Creeping Matrix

#### Abstract

Ceramic matrix composites (CMC) are potentially designed for use in the high temperature environment because of their high strength and noncatastrophic failure characteristics. For ceramics at elevated temperature, the materials exhibit time-dependent deformation. The phenomenon is further accelerated by an increase in the stress or the temperature. Reinforcement by incorporating high-strength fibers is one of the several approaches to significantly improving the creep fracture resistance of the CMC. Experimental studies in mechanical behavior of CMC [1-4] have shown that failure is preceded by matrix cracking at high temperatures. Further, the creeping matrix causes stress transfer from the matrix to the fiber. However, there is a lack of any investigation in understanding of the relation between the matrix cracking at the micro level and the overall mechanical behavior of CMC at elevated temperature.

In this research effort, the crack tip fields for a matrix crack are considered. Using generalized expansions at the crack tip in each region and matching the stresses and displacements across the interface in an asymptotic sense, a series asymptotic solution is constructed for the stresses and strain rates near the crack tip. It is found that the stress singularities, to the leading order, are the same in each material, oscillatory higher-order terms exist in both regions, and stress higher-order term with the order of  $O(r^0)$  appears in the elastic material. The stress exponents and the angular distributions for singular terms and higher order terms are obtained for different creep exponents. A full agreement between asymptotic solutions and the full-field finite element solutions has been obtained.

#### Asymptotic Analysis

Crack tip fields of a matrix crack in an elastic power-law creep material perpendicular to an interface separating elastic material shown in Fig. 1 are investigated. The constitutive equation for elastic material 1 can be expressed by

$$\dot{\varepsilon}_{ij} = \frac{1+\nu_1}{E_1} \dot{s}_{ij} + \frac{1-2\nu_1}{3E_1} \dot{\sigma}_{kk} \delta_{ij} \quad (1)$$

The constitutive equation for the elastic power-law creeping material 2 is given by

$$\dot{\varepsilon}_{ij} = \frac{1+\nu_2}{E_2} \dot{s}_{ij} + \frac{1-2\nu_2}{3E_2} \dot{\sigma}_{kk} \delta_{ij} + \frac{3}{2} \dot{\varepsilon}_0 \left( \frac{\sigma_e}{\sigma_0} \right)^{n-1} \frac{s_{ij}}{\sigma_0} \quad (2)$$

where  $\dot{\varepsilon}_0$  and  $n > 1$  are material constants;  $\sigma_0$  is a reference stress. Note that  $\dot{\varepsilon}_0$  is allowed to be a function of time  $t$  in modeling transient creep or material aging.

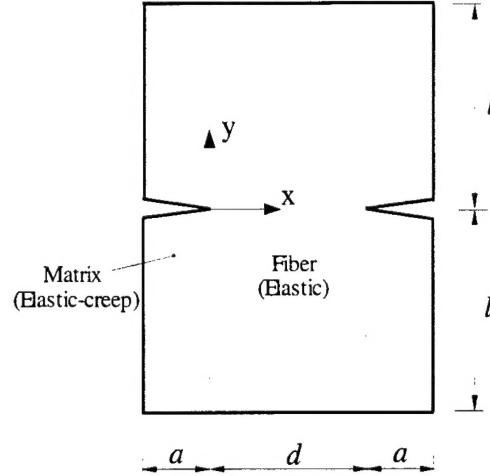


Fig. 1 Geometry of the cracked unit cell

Referring to a polar coordinate system with origin centered at the crack tip, as  $r \rightarrow 0$ , in creeping material we seek an asymptotic expansion of the stress function  $\Phi_2$  in a form

$$\Phi_2(r, \theta; t) = A_0(t)r^{q_0+2}f_0(\theta; q_0) + \sum_{k=1} \operatorname{Re}[A_k(t)r^{q_k+2}f_k(\theta; q_k)] \quad (3)$$

where

$$q_0 < \operatorname{Re}(q_1) < \operatorname{Re}(q_2) < \dots$$

To ensure the existence of the solution in eq. (3),  $q_0$  must be real and  $q_k$  ( $k \geq 1$ ) may be complex. Thus  $f_0$  and  $A_0$  are real,  $f_k$  and  $A_k$  may be complex. For plane strain problem, the stress component  $s_{zz}$  can not be expressed directly in terms of  $\Phi$ . we further assume the deviatoric stress  $s_{zz}(r, \theta; t)$  can be expanded as

$$s_{zz} = A_0 r^{q_0} s_{zz}^{(0)}(\theta) + \sum_{k=1} \operatorname{Re}[A_k r^{q_k} s_{zz}^{(k)}(\theta)] \quad (4)$$

The corresponding expansions of in-plane stress tensor  $\sigma$ , deviatoric stress tensor  $s$ , stress rate  $\dot{\sigma}$ , the effective stress  $\sigma_e$ , strain rate  $\dot{\varepsilon}$ , and displacement rate  $\dot{u}$  can be obtained.

Therefore, the expansions of the compatibility equation, plane strain condition,  $\dot{\varepsilon}_{zz} = 0$ , can be expressed in terms of  $f_k$  and  $s_{zz}^{(k)}$ .

In the region of linear elastic material, the expansion of stress function  $\Phi_1$  is written as

$$\Phi_1(r, \theta; t) = B_0(t)r^{t_0+2}F_0(\theta; t_0) + \sum_{k=1} \operatorname{Re}[B_k(t)r^{t_k+2}F_k(\theta; t_k)] \quad (5)$$

The solutions for symmetric loading by satisfying a compatibility equation are

$$F_k = c_k \cos t_k \theta + d_k \cos(t_k + 2)\theta \quad (6)$$

where  $c_k$  and  $d_k$  are constants to be determined.

The boundary conditions for this problem are

$$(\sigma_{\theta\theta}^{(k)})_2 = (\sigma_{r\theta}^{(k)})_2 = 0, \quad \text{at } \theta = \pi \quad (7)$$

The continuity conditions along the interface,  $\theta = \pi/2$ , are

$$(\sigma_{\theta\theta})_2 - (\sigma_{\theta\theta})_1 = 0, \quad (\sigma_{r\theta})_2 - (\sigma_{r\theta})_1 = 0, \quad (\dot{u})_2 - (\dot{u})_1 = 0 \quad (8)$$

## Asymptotic Fields

Collecting the coefficients of like power of  $r$  in the expansions of compatibility equation,  $\varepsilon_z = 0$ , boundary conditions and continuity conditions, equating the coefficient of each power of  $r$  to zero, the governing equations for each order can be obtained. It is found that the leading order term has identical stress exponent,  $q_0 = t_0$ , in both regions, and higher-order solutions can be categorized into two distinct types. The first type solution is determined by a homogeneous ordinary differential equation with homogeneous boundary condition on the interface, which leads to linear eigenvalue problems. The other is governed by the nonhomogeneous equations. Comparing and then selecting the minimum value of the stress exponents decided by the eigenvalue (homogeneous) problem and by the nonhomogeneous problems, we can determine  $q_p, t_1, q_1, t_2, \dots$ , etc. Fig. 2(a) shows variations of  $q_i$  ( $i = 0, 1, 2, 3$ ) with  $n$  for  $1 < n < 20$ . By inserting the zero stress exponent in the figure, the variations of  $t_k$  ( $k = 0, 1, 2, 3, 4$ ) can be directly obtained.

Solving the governing equations, the stress exponents and angular distribution for the leading order and higher order terms can be obtained. The multi-term expansion of the crack-tip stress fields are given. For example, for  $1.5 < n < 2$

$$\begin{aligned} (\sigma)_2 &= A_0 r^{q_0} \sigma^{(0)} + A_1 r^{(2-n)q_0} \sigma^{(1)} + r^{(3-2n)q_0} \left( A_0^{-1} A_1^2 \sigma^{(2)I} + \alpha^{-1} A_0^{1-n} \dot{A}_1 \sigma^{(2)II} \right) \\ &\quad + \alpha^{-1} A_0^{1-n} \dot{B}_2 r^{(1-n)q_0} \sigma^{(3)} \\ (\sigma)_1 &= A_0 r^{q_0} \sigma^{(0)} + A_1 r^{(2-n)q_0} \sigma^{(1)} + B_2 \sigma^{(2)} + r^{(3-2n)q_0} \left( A_0^{-1} A_1^2 \sigma^{(3)I} + \alpha^{-1} A_0^{1-n} \dot{A}_1 \sigma^{(3)II} \right) \\ &\quad + \alpha^{-1} A_0^{1-n} \dot{B}_2 r^{(1-n)q_0} \sigma^{(4)} \end{aligned}$$

where  $A_0(t)$  and  $B_2(t)$  are real arbitrary amplitudes.  $A_1 = A_0^{1-n} \dot{A}_0 / \alpha$ .

## Discussion and Conclusions

A unit cell in fiber reinforced composite with matrix cracks under plane strain shown in Fig.1 has been analyzed. Uniform displacement is applied to the end of the cell with geometry of  $d/a = 4$  and  $l/d = 5$  parallel to the fiber axis at  $t > 0$ . Under symmetric loading, the boundary conditions for the model are:

$$\begin{aligned} u_y(t) &= \pm l \varepsilon(t), \quad \sigma_{xy} = 0 \quad \text{at } y = \pm l \\ u_x &= 0, \quad \sigma_{xy} = 0 \quad \text{at } x = -a, d/2 \end{aligned}$$

where

$$\varepsilon(t) = \varepsilon_\infty \left\{ 1 - \left[ 1 + \frac{(n-1)\alpha}{\varepsilon_\infty} t \right]^{-1/(n-1)} \right\}, \quad \varepsilon_\infty = \frac{\sigma_c}{E_f V_f}, \quad \alpha = \frac{\dot{\varepsilon}_0}{\sigma_0^n} \left( \frac{\sigma_c}{V_m} \right)^n \left( 1 + \frac{E_f V_f}{E_m (1 - V_f)} \right)^{-1},$$

$V_f$  is the volume fraction of the composite. The matrix material parameters are  $E = 100$  GPa,  $\nu = 0.2$ ,  $n = 1.8$ ,  $\sigma_c/E = 0.002$ ,  $\dot{\varepsilon}_0 = 0.002 / hr$  and  $\sigma_0 = \sigma_c$ . The fiber material constants are  $E = 350$  GPa,  $\nu = 0.2$ . In this case, the stress exponents are given by

$$\begin{aligned} q_0 &= -0.2940, \quad q_1 = (2-n)q_0, \quad q_2 = (3-2n)q_0, \quad q_3 = (1-n)q_0 \\ t_0 &= q_0, \quad t_1 = (2-n)q_0, \quad t_2 = 0, \quad t_3 = (3-2n)q_0, \quad t_4 = (1-n)q_0 \end{aligned}$$

Numerical computation is implemented in the ABAQUS finite element code. Eight-node isoparametric elements with  $2 \times 2$  Gauss integration points are used. The size of the smallest crack tip elements is  $1.6 \times 10^{-5}a$ . Since the time-dependent amplitudes in the asymptotic expansion are unknown, the values of the amplitudes are determined by matching asymptotic

solution with finite element solution. Once the values are determined, the entire asymptotic crack-tip fields are known. The matching is forced  $\sigma_r$  or/and  $\sigma_\theta$  at  $\theta = 0$  or  $\pi/2$  in the fiber. Results show that the values of the unknown amplitudes determined from different stress components at various locations vary slightly.

Fig. 2(b) indicates the radial distribution of  $\sigma_{\theta\theta}$  along  $\theta = 0$  at  $t = 0.06/\dot{\varepsilon}_o$ . Here  $\sigma_\theta$  is normalized by  $\sigma_0$ . Figs. 2(c) and 2(d) show that the angular variations at  $r/a = 2.312 \times 10^{-3}$  and  $t = 0.06/\dot{\varepsilon}_o$ . When  $t = 0.2/\dot{\varepsilon}_o$  and  $r/a = 1.8 \times 10^{-3}$ , the angular distributions are shown in Figs. 2(e) and 2(f). Good agreement is presented in these figures. Comparison of the asymptotic solutions to finite element solution indicates that the asymptotic solutions can describe the crack-tip field over a significant scale.

Analysis demonstrates that for short time after application of the loading, several terms are needed to characterize the crack-tip fields. As  $t \rightarrow \infty$ , stress field in elastic-creeping material can be characterized by the leading-order singular term within a substantial distance from the crack-tip; while in elastic material two parameters including the amplitude of the constant stress term may be needed.

In this paper, the generalized separable solutions for the stress functions  $\Phi_i$  are used which lead to multi-term expansion of crack-tip fields. Although the leading order solutions do not have oscillatory nature, the oscillatory behavior may prevail in the higher-order solutions. Asymptotic analyses indicate that the leading term is characterized by a real parameter  $A_o(t)$ . In the creeping material, when  $n < 7$ , the second-order term is controlled by both elasticity and creep and characterized by  $A_o$ ; when  $n > 7$ , the complex stress exponent appears in the second term which is described by a complex parameter. In the elastic material, there exists constant stress field with order  $r^0$ , this term constitutes the second order solution for the range  $2 < n$ ; the complex stress exponent enters the third term for  $7 < n < 20$ . The analysis can be extended to the cracks on the interfaces between two elastic-power-law creep materials ( $n_1 > n_2 \geq 1$ ).

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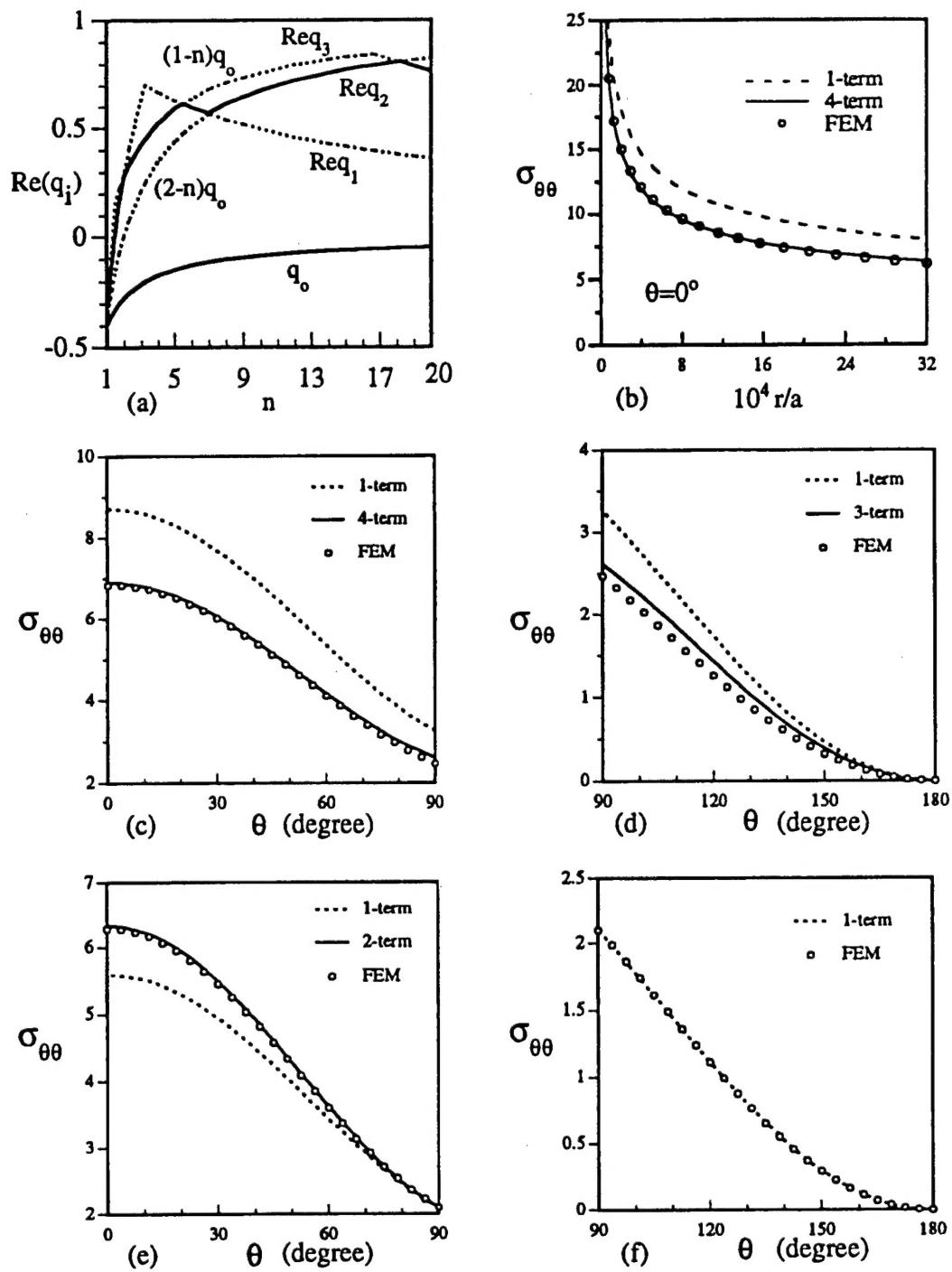


Fig. 2 (a) Variation of the stress exponents  $q_i$  with  $n$  in elastic-creeping material  
 (b) Radial distributions of stress  $\sigma_{\theta\theta}$  at  $\theta=0$  and  $t=0.06/\dot{\varepsilon}_0$   
 (c) and (d) Angular distributions of stress  $\sigma_{\theta\theta}$  at  $r/a=2.312 \times 10^{-3}$  and  $t=0.06/\dot{\varepsilon}_0$   
 (e) and (f) Angular distributions of stress  $\sigma_{\theta\theta}$  at  $r/a=1.8 \times 10^{-3}$  and  $t=0.2/\dot{\varepsilon}_0$

## (B) Re-examination of Microstress in a Model Composite Laminate

### Abstract

Failure modes of composite laminates have their origins in the micromechanical domain. The analysis of failure in fiber composites has traditionally followed two distinct levels of abstraction. The areas of investigation are known as *micromechanics* and *macromechanics*:

The micromechanics approach aims at the involvement of microscopic inhomogeneities in various kinds of micro-failure processes by taking the composite microstructure into account. The advantage of the micromechanics representation is that detailed information is directly obtained about the local interaction between the constituents and micro-failure mechanisms (Yuan and Yang, 1997). The numerical modeling of exceedingly complicated geometric detail of each fibers and matrix, however, often requires exceedingly fine grids and hence results in excessive computer cost. It is obvious that conducting a stress analysis for composite laminates using this approach is an almost impossible task even beyond the computational capacity of the latest supercomputers. Hence, the micromechanical model is mainly restricted to the strength prediction in the lamina level or unidirectional composites.

Since the inclusion of all inhomogeneities in a mechanical model of laminates is an overwhelming task, in the macromechanics approach, the details of the composite microstructure are ignored and the composite is treated as a "locally" homogeneous (it may be inhomogeneous in the laminate level) anisotropic material. The homogeneous anisotropic constitutive relationships are determined from micromechanics analysis of homogenization via continuum mechanics. In this so called effective-modulus theory, the volume averaged stresses are related to volume averaged strains by means of effective elastic moduli. Implicit in the definition of the anisotropic material properties is the existence of uniform macroscopic state of stress (Pagano, 1974). Since the size of the representative element may be larger or comparable to the region where the stress concentrations are dominant, it is hard to justify the validity of such results. Further because the microstructure is ignored, the detailed localized influence of the geometric features simply does not exist in these models. Thus, the resulting analysis is empirical and phenomenological. Although the final equations may have some predictive value and may be used for engineering design, they do not give insight into any microstructural aspect of failure.

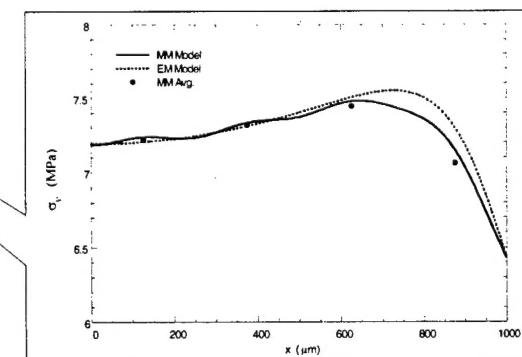
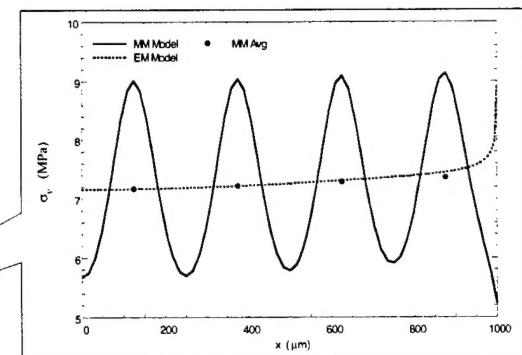
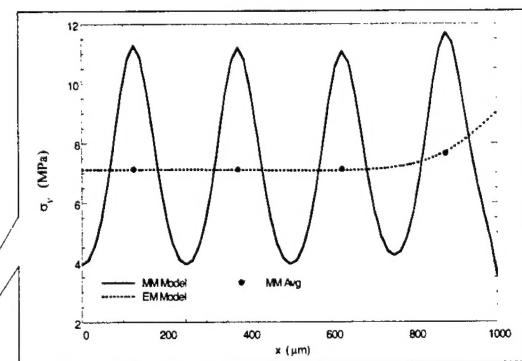
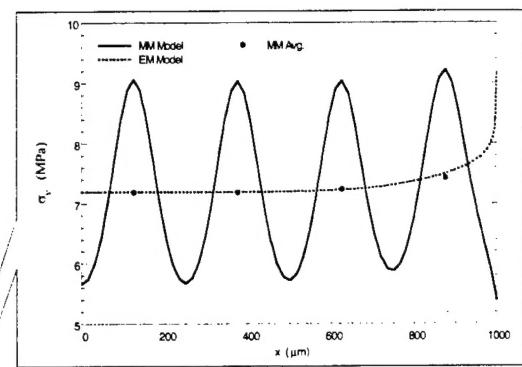
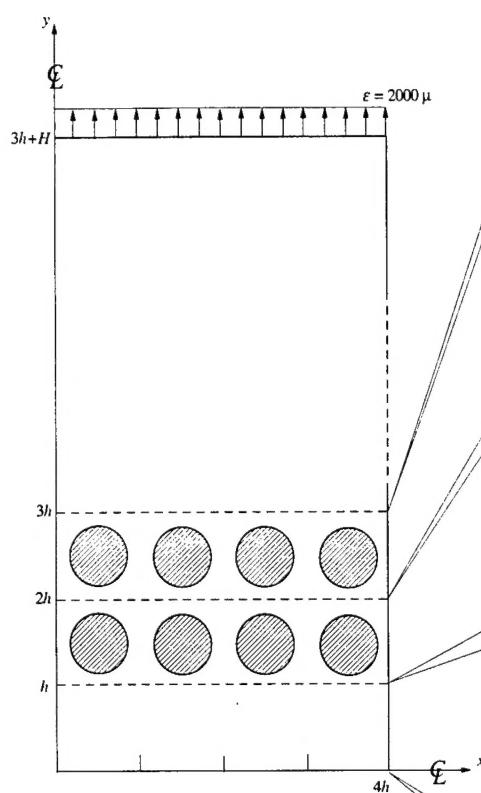
From the existing analytical approaches, it is clear that micromechanics approach alone will not explain the failure process of the laminates simply because the mutual interaction between the micro-stresses and interlaminar stresses in the failure process has been totally neglected. While in the macromechanics approach, although the macroscopic or overall constitutive descriptions are developed from composite microstructure in terms of the volume fraction, the shape, and the interface conditions of the constituents, the constitutive relations are independent of the scale of the microstructure. Further, the effective-modulus theory, in principle, only applies to macroscopically uniform fields. Therefore, the stress fields near the high stress gradient regions using the classical approximation are questionable (Pagano and Rybicki, 1974 and Rybicki and Pagano, 1975). Since the details of the generally

complex, strongly heterogeneous microstructures are not considered directly, the conventional macroscopic theory would inevitably involve erroneous theoretical predictions, and from which precise information of failure in the micro-level would not be possible to elicit.

Apparently, in regions of macroscopically steep stress (or strain) gradients such as free edges or holes, the conventional anisotropic elasticity theory utilizing the effective-modulus concept in the constitutive relations does not preserve the essential variation of the microstress distribution through the unit-cell in the micro-level. Indeed, rapid change of three-dimensional stress state contradicts the underlying assumption of the macroscopically uniform state of stress from which the effective-modulus theory has been derived. As a consequence, at high stress gradient regions the conventional approach ceases to provide true representation of physical reality. In order to represent the effect of the microstress variation through the unit cell and properly capture the meaningful macroscopic steep stress gradient fields, one must retain the volume average of the microstress distribution as the 'stress' acting on the cell but also the moment of the microstress distribution, termed couple stress, on the element.

### Numerical Analysis

In this work, we consider the distribution of the phase material stresses within a simulated free edge specimen. The fibrous layers are all oriented in the longitudinal or loading direction, while the remaining layers are entirely composed of matrix material. This analog was first treated by Pagano and Rybicki (1974). Finite element analyses shown in the figure are performed under generalized plane deformation. In the present case, we compute the phase stresses, not only along the (artificial) interfacial boundaries and free edge boundary, but along the entire boundaries of the unit cells surrounding each fiber. It is shown that these stresses lead to resultant effects in which both the forces and moments are non-zero. Thus, as shown in the earlier work, we demonstrate that the effective modulus solution, in which the composite is modeled by a system of layers, each of which is assumed to have constant elastic moduli, is not valid. This may present significant implications on the prediction of initial damage in the body as both the location and character of the initial damage differs in the two problems. This, in turn, is shown by examining the interfacial stresses, which may precipitate initial cracking in various ways depending on the stacking sequence, character of applied loading, and constituent material properties. The severity of the errors in the usual effective modulus (ply elasticity) approach are evaluated by conducting parametric studies of the material and geometric variables. Consequences on the prediction of initial failure by incorporating thermal residual stresses are considered.



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## III. Personnel Supported

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Research Associate: Dr. S. Yang

## IV. Publications

1. F. G. Yuan and S. Yang, "Crack Tip Fields of Matrix Cracks between Dissimilar Elastic and Creeping Materials, Submitted to *International Journal of Fracture*, February, 1999.
2. N. J. Pagano and F. G. Yuan, "Microstress Fields in a Simulated Free Edge Specimen", Presented in *ICE5 Meeting*, Las Vegas, July, 1998.
3. N. J. Pagano and F. G. Yuan, "Potential Effects of Micromechanical Stresses on the Failure Initiation in Composite Laminates", *Presented in the ASME Winter Meeting*, Anaheim, CA, November, 1998.
4. W. X. Wang, Y. Takao, F. G. Yuan et al. "The Interlaminar Mode I Fracture of IM7/LaRC-RP46 Composites at High Temperatures", *Journal of Composite Materials*, Vol. 32, No. 16, pp. 1508-1526, 1998.
5. F. G. Yuan and P. Hutzapea, "Mode-I Fracture Toughness of IM7/LaRC-RP46 Composites under Thermal Aging", *39<sup>th</sup> AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, April 20-23, 1998, Long Beach, CA.
6. F. G. Yuan and S. Yang "Crack-Tip Fields in Elastic-Plastic Material under Plane Stress Mode I Loading", *International Journal of Fracture*, Vol. 85, pp. 131-155, 1997.

7. F. G. Yuan and P. Hutaapea, "The Mode-I Fracture Toughness of IM7/LaRC-RP46 Composites at High Temperature", International Conference of Composite Materials, Goldcoast, Australia, July, 1997.
8. S. Yang and F. G. Yuan, "Matrix Cracking Analysis in Composites", 38th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Material Conference, April 7-10, 1997, Kissimmee, FL.
9. F. G. Yuan and S. Yang, "Crack Tip Fields for Cracks Perpendicular to the Interface between Dissimilar Elastic and Creeping Materials" ASME Mechanics and Materials Conference, The John Hopkins University, Baltimore. June 12-14, 1996.
10. S. Yang and F. G. Yuan, "Crack-Tip Fields in Elastic-Plastic material under Mode I Plane Stress Loading", 1996 ASME International Mechanical Engineering Congress and Exposition, November 17-22, 1996, Atlanta, Georgia.

#### V. Interactions/Transitions

The principal investigator has been working closely with Dr. N. J. Pagano, Materials Laboratory at WPAFB, on the technical issues of microcracking of brittle matrix composites. The Materials Laboratory has been tested several simulated free edge specimens under tension normal to the fiber direction. Failure scenarios of the failure process will be quantified. Currently, Dr. Yuan has been trying to model these fracture processes using analysis methods and predict the failure processes by altering the material and interface strength. The most dominant singular term of the stress field and associated angular distributions have been obtained. Further study will be continued to determine the failure initiation.